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**Thermosolutal Instability of Rivlin-Ericksen Rotating Fluid in Porous Medium using Brinkman Model**

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**Abstract-**

The paper examines thermosolutal instability of Rivlin-Ericksen rotating fluid in porous medium using Brinkman model and have shown that more instability is expected when Brinkman model is considered.

**Keywords:-** Rivlin-Ericksen fluid, Porous medium, Brinkman model.

**1. Introduction:**

The theoretical and experimental results on the onset of thermal instability (Benerd Convection) in a fluid layer under varying assumptions of hydrodynamics have been treated in detail by **Chandrasekhar** (1981) in his celebrated monograph. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by **Veronis** (1965).

With the growing importance of Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. The **Rivlin-Ericksen** (1955) fluid is one such fluid. Many research workers have paid their attention towards the study of Rivlin-Ericksen fluid. **Johri** (1976) has discussed the visso-elastic Rivlin-Ericksen incompressible fluid under the time-dependent pressure gradient. **Sisodia** and **Gupta** (1984) and **Srivastava** and **Singh** (1988) have studied the unsteady flow of a dusty elastic-viscous Rivlin-Ericksen fluid through channel of different cross section in the presence of the time-dependent pressure gradient. **Sharma** and **Kumar** (1996) have studied the thermal instability of a layer of Rivlin-Ericksen elastico-viscous fluid acted on by a uniform rotation and found that rotation has a stabilizing effect and introduces oscillatory modes in the system.

In many astrophysical situations, the effect of rotation on porous medium is also important and has been discussed by **Sharma *et al.*** (1997).

Keeping in mind the importance in geophysics, soil physics, astrophysics, ground water hydrology and various applications mentioned above, the thermosolutal instability of a Rivlin-Ericksen fluid in porous medium in the presence of uniform verticals rotation using Brinkman's model has been considered.

**2. Formulation of the Problem and Perturbation Equations:**

Here we consider an infinite, horizontal, incompressible, Rivlin-Ericksen fluid layer of thickness d, heated and soluted from below so that, the temperatures, densities and solute concentrations at the bottom surface z = 0 are To, ρ0, and C0 and at the upper surface z = d are Td, ρd, and Cd, respectively,

and that a uniform temperature gradient  and uniform solute gradient  are maintained. The gravity field **g** (0, 0, -g), and a uniform vertical rotation w (0, 0, Ω) pervade the system. The fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ε and medium permeability k1.

Let p, ρ, T, C, α, α' **g** and **q** (u, v, w) denote respectively the fluid pressure, density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, gravitational acceleration and the fluid velocity. The equations expressing the conservation of momentum, mass, temperature, solute mass concentration and the equation expressing δ in terms of temperature and concentration are:



 (1)

∇.**q** = 0, (2)

 + (**q.** ∇) T = κ∇2 T, (3)

 (4)

And ρ = ρ0 [1 – α (T – T0) + α' (C – C0)], (5)

Where the suffix zero refers to the values at the reference level z = 0 and in writing equation (1), use has been made of the Boussine'sq approximation. The kinematic viscosity ν, kinematic viscoelasticity ν', the thermal diffusivity κ and the solute diffusivity κ' are all assumed to be constant. E=ε + (1-ε) (ρs Cs/ρ0 Ci) is a constant and E' is a constant analogous to E but corresponding to solute rather than heat ρs, Cs; ρ0 and Ci denote the density and heat capacity of solid (porous matrix) material and fluid respectively.

The steady state solution is-

**q** = (0, 0, 0), T = ‒βz + T0, (6)

C = ‒β' z + C0, ρ = ρ0 (1 + αβz – α'β' z)

Here we use linearised stability theory and normal mode analysis method. Consider a small perturbation on the steady state solution and let δp, δρ, θ, γ and **q** (u, v, w) denote respectively, the perturbation in pressure p, density ρ, temperature T, solute concentration C and velocity **q** (0, 0, 0). The change in density δρ, caused mainly by the perturbations θ and γ in temperature and solute concentration is given by-

δρ = ‒ρ0 (αθ – α'γ), (7)

Then the linearised perturbation equations become



+ (8)

∇.**q** = 0, (9)

 (10)

and  (11)

**3. The Dispersion Relation:**

Analysing the disturbances into normal modes, we assume that the perturbation quantities are of the form

[w, θ, γ, ζ] = [W (z), Θ (z), Γ (z), Z(z)] exp (ikx *x* + iky y + nt)

Here kx and ky are the real wave numbers in the x and y-directions.

k =  and n (= nr + ini) is complex, in general. Now substituting the above expression in the equations (8) – (11), he have



+  (12)

E n Θ = βw + κ (D2 – k2) Θ, (13)

E n γ = β' w + κ' (D2 – k2) Γ, (14)

And if the rotation is in the vertical direction, the relevant equation is

 (15)

Now expressing the coordinates x, y, z in the new unit of length d and letting  we get the equation (12)-(15), in the non-dimensional forms as-



 (16)

 (17)

[D2 – a2 – E' qσ] Γ = ‒  (18)

and  (19)

Now eliminating Θ, Γ and z from equation (16)-(19), we get the final stability governing equation as-



(D2 – a2 – E' qσ) (D2 – a2) w + 

(D2 – a2 – E' qσ) 

(D2 – a2 – Ep1σ) 

 (20)

Consider the case where both boundaries are free as well as maintained at constant temperature and solute concentrations. The appropriate boundary conditions are-

w = D2w = ο, Θ = 0, Γ = 0, Dz = 0 at z = 0 and 1 (21)

The case of two free boundaries is a little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. Let the proper solution of w characterizing the lowest mode is-

w = w0 Sin π z, (22)

Where w0 is a constant

Now substituting the proper solution w=w0 Sinπz in (20), we obtain the dispersion relation

 (23)

where R1 =  (24)

and

Equation (23) is the required dispersion relation for studying the effect of rotation, medium permeability, kinematic visco-elasticity and stable solute gradient on thermosolutal instability of Rivlin-Ericksen rotating fluid in porous medium by using the Brinkman model.

**4. Stationary Convection:**

When the instability sets in as stationary convection, the marginal state will be characterized by σ = 0 putting σ = 0, the dispersion relation (23) reduces to

+S1, (25)

Which gives the modified Rayleigh number and shows that the effect of visco-elasticity disappears for the stationary convection.

Also equation (25) yields



This shows that the relation has always a stabilizing effect on the thermosolutal instability of Rivlin-Ericksen rotating fluid in porous medium.

Also from equation (25)





As 

, which is always negative.

Hence more instability is expected when Brinkman's model is considered.

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